Mathematical Conceptualisation of Shooting Down a Drone/Helicopter

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The objective of this commentary is to determine several parameters of the shooting down of slow flying objects.[1] If we have some basic information regarding the motion of the drone/helicopter and we also know the velocity with which the bullet will be discharged, then we may evaluate the angle of projection (α) so that the bullet will shoot down the drone/helicopter even if it has manoeuvring effect due to acceleration or retardation. Obviously, the conclusions of the commentary will also hold if any other objects in the air replace the drone/helicopter. The reason that we have specifically concentrated on the drone/helicopter is because it is practically possible for a bullet to strike it. In the case of fighter jets, which usually move at great speed, further incentive momentum would be required during the motion to enable a striking projectile to make contact with them in the available time. This is possible for a guided missile, but not a bullet.

Suppose a bullet is fired with some force so that at the time of discharge its initial speed u follows a direction that makes angle with the horizontal axis. Now, the bullet will strike a drone/helicopter in air if two empirical conditions are satisfied:

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ISSN 0976-1004 print

Journal of Defence Studies, Vol. 16, No. 4, October–December 2022, pp. 263–274

(a) The horizontal location of the flying object must be coincident with the horizontal location of the shell at the same time (t).
(b) The vertical location (height) of the flying object must be coincident with the vertical distance travelled by the shell at the same time (t).

Keeping in view these two empirical conditions, we will formulate the master equations for shell trajectory to hit flying objects such as drones/helicopters in the next section. The trajectory of the bullet is a parabola having the equation[2]

$$y = x \tan \alpha - \frac{(gx^2)}{(2u^2 \cos^2 \alpha)} \qquad ...(1)$$

The commentary is planned as follows: The next section consists of the mathematical formation of the master equation with analysis of various parameters while the drone/helicopter is moving away from the observer. In the subsequent section, with a study of several parameters, we derive the master equation while the drone/helicopter is moving towards the observer. The last two sections cover conclusion remarks and provide the summary of the commentary.

Formulation of the Master Equation While the Drone/Helicopter is Moving Away from the Observer with Acceleration

Let O be any point of projection of the shell and A be the position of the drone/helicopter when the shot was fired. The muzzle velocity (initial velocity) of the shell is along OD, let the angle of projection be α. The

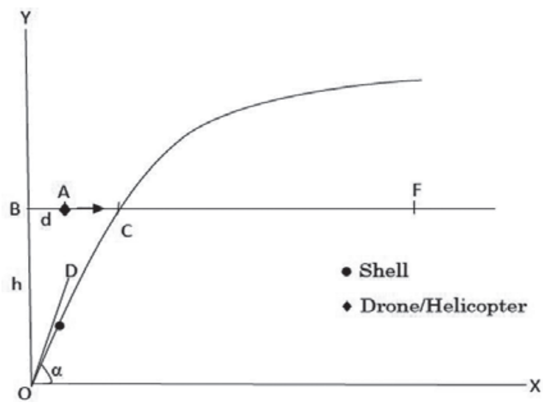

Figure 1 The Drone/Helicopter is Moving Away from the Observer with Acceleration *f*
Source: Authors' own.

drone/helicopter is flying at a height of h meters along the line AF with an initial observed speed $\mathbf{v_0}$ **m/s** and acceleration **f m/s²**. C[3] is a point where the shell can hit the drone/ helicopter, that is, provided both reach C simultaneously (Figure 1).[4] The starting point of the shell is O, and at the same time the position of the flying object is A such that BA = d.

Let the shell hit the drone/helicopter after time t.

The distance moved by the **drone** in time t

$$AC = v_0.t + \tfrac{1}{2} ft^2 \qquad ...(2)$$

Horizontal distance moved by the shell after time t is given by

$$BC = u \cos \alpha.t \qquad ...(3)$$

In view of Equations (2) and (3), we have

$$BC = d + AC \Rightarrow u \cos \alpha.t = d + (v_0.t + \tfrac{1}{2} ft^2)$$

$$\tfrac{1}{2} ft^2 + (v_0 - u \cos \alpha).t + d = 0 \qquad ...(4)$$

and yields the condition

$$t = \frac{(u \cos \alpha - v_0) - \sqrt{(u \cos \alpha - v_0)^2 - 2\, fd}}{f} \text{ if } f \neq 0 \qquad ...(5)$$

$$= \frac{d}{(u \cos \alpha - v_0)} \text{ if } f = 0 \qquad ...(6)$$

Vertical motion of the shell from O to C is given by

$$h = u \sin \alpha.t - \tfrac{1}{2} gt^2 \qquad ...(7)$$

Using Equations (5), (6) and (7), we get

$$4.905 \left(\frac{(u \cos \alpha - v_0) - \sqrt{(u \cos \alpha - v_0)^2 - 2\, fd}}{f}\right)^2 - \frac{(u \cos \alpha - v_0) - \sqrt{(u \cos \alpha - v_0)^2 - 2\, fd}}{f}\, u \sin \alpha + h = 0$$

$$\text{if } f \neq 0 \qquad ...(8)$$

$$4.905 \left(\frac{d}{(u \cos \alpha - v_0)}\right)^2 - \frac{d\, u \sin \alpha}{(u \cos \alpha - v_0)} + h = 0 \text{ if } f = 0 \qquad ...(9)$$

The angle of projection can be obtained by using Equations (8) and (9).

Evaluation of Various Requisite Parameters

We calculated various parameters such as the angle of projection, time of hit and distance covered by the drone/helicopter at the time of hit by considering the speed of drone as 20m/s, speed of bullet as 900m/s and detected that the drone/helicopter is flying at a height of 500m, horizontal distance of 800m and that it is moving away from the observer through the master equations developed by us (see Tables 1, 2 and Figures 2 and 3). Through the master equations, we also estimated the same parameters for various values of height (h = 900m to 1500m) of a drone and a helicopter for f = 0.5m/s^2 and f = 2.5m/s^2 respectively (Tables 3, 4 and Figures 4 and 5) as well as established when the flying objects are moving away from the observer. The trends of time taken to hit the drone/helicopter (T), angle of projection (α) and distance travelled by the drone/helicopter during the time of hit (AC) can be seen in Figures 2, 3, 4, and 5.

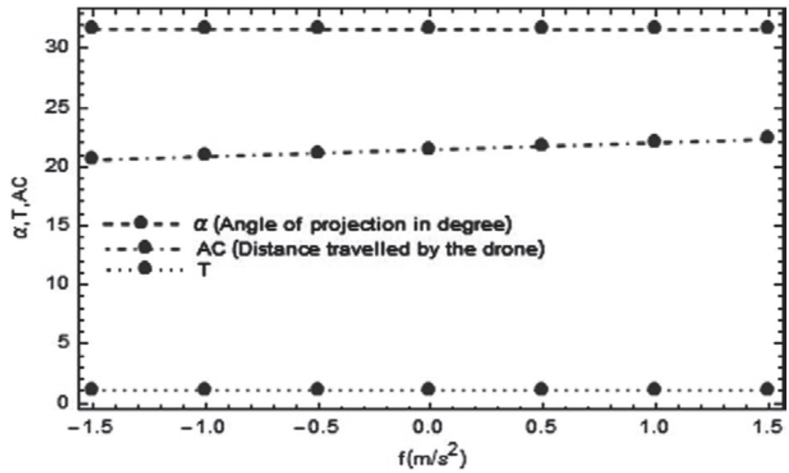

Figure 2 Variation of α, T, AC with f while the Drone is Moving Away from the Observer when h and d are Fixed
Source: Authors' own.

Table 1 Values of the Angle of Projection, Time of Hit and Distance Covered by the Drone Moving Away from the Observer during the Time of Hit when h and d are Fixed

S. No.	*Speed of Drone m/s (v_0)*	*Acceleration/ Retardation f m/s^2*	*Muzzle Velocity of Weapon m/s u*	*Horizontal Distance in Metre d*	*Height of Drone h*	*Time Taken to Hit the Drone T*	*Angle of Projection in Degree*	*Distance Travelled by the Drone during Time of Hit (AC)*
1	20	–1.5	900	800	500	1.07093	31.6412	20.5583
2	20	–1	900	800	500	1.0712	31.6323	20.8503
3	20	–0.5	900	800	500	1.07148	31.6233	21.1426
4	20	0	900	800	500	1.07176	31.6144	21.4352
5	20	0.5	900	800	500	1.07204	31.6054	21.7281
6	20	1	900	800	500	1.07232	31.5964	22.0213
7	20	1.5	900	800	500	1.0726	31.5874	22.3148

Source: Authors' own.

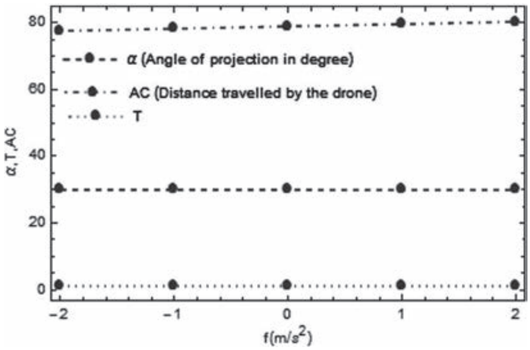

Figure 3 Variation of α, T, AC with *f* While the Helicopter is Moving Away from the Observer When *h* and *d* are Fixed
Source: Authors' own.

Table 2 Values of the Angle of Projection, Time of Hit and Distance Covered by the Helicopter Moving Away from the Observer during the Time of Hit When h and d are Fixed

S. No.	*Speed of Helicopter m/s (v_0)*	*Acceleration/ Retardation f m/s²*	*Muzzle Velocity of Weapon m/s u*	*Horizontal Distance in Metre d*	*Height of Helicopter h*	*Time Taken to Hit the Helicopter T*	*Angle of Projection in Degree*	*Distance Travelled by the Helicopter during Time of Hit (AC)*
1	70	–2	900	800	500	1.12563	29.9792	77.5272
2	70	–1	900	800	500	1.12629	29.9603	78.2059
3	70	0	900	800	500	1.12695	29.9415	78.8864
4	70	1	900	800	500	1.12761	29.9226	79.5683
5	70	2	900	800	500	1.12827	29.9038	80.252

Source: Authors' own.

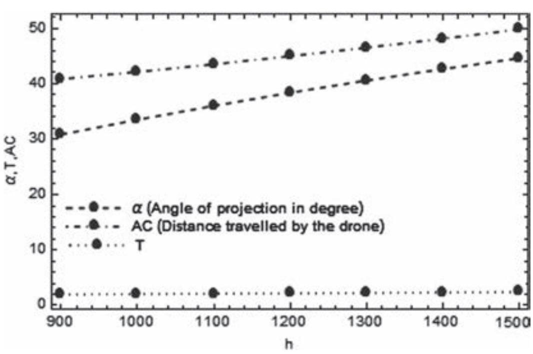

Figure 4 Variation of α, T, AC with *h* While the Drone is Moving Away from the Observer When *f* and *d* are Fixed
Source: Authors' own.

Table 3 Values of the Angle of Projection, Time of Hit and Distance Covered by the Drone Moving Away from the Observer during the Time of Hit When *d* and *f* are Fixed

S. No.	*Speed of Drone m/s (v_0)*	*Acceleration/ Retardation f m/s²*	*Muzzle Velocity of Weapon m/s u*	*Horizontal Distance in Metre d*	*Height of Drone h*	*Time Taken to Hit the Drone T*	*Angle of Projection in Degree*	*Distance Travelled by the Drone during Time of Hit (AC)*
1	20	0.5	900	1500	900	1.99374	30.8262	40.8686
2	20	0.5	900	1500	1000	2.05483	33.4995	42.1522
3	20	0.5	900	1500	1100	2.1203	36.0148	43.5299
4	20	0.5	900	1500	1200	2.18977	38.3766	44.9941
5	20	0.5	900	1500	1300	2.26288	40.5909	46.5378
6	20	0.5	900	1500	1400	2.33932	42.66491	48.1545
7	20	0.5	900	1500	1500	2.41878	44.6065	49.8383

Source: Authors' own.

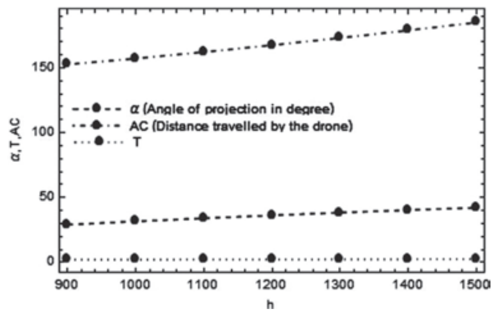

Figure 5 Variation of α, T, AC with *h* While the Helicopter is Moving Away from the Observer When *f* and *d* are Fixed
Source: Authors' own.

Table 4 Values of the Angle of Projection, Time of Hit and Distance Covered by the Helicopter Moving Away from the Observer during the Collision When *f* and *d* are Fixed

S. No.	*Speed of Helicopter m/s (v_0)*	*Acceleration/ Retardation f m/s²*	*Muzzle Velocity of Weapon m/s u*	*Horizontal Distance in Metre d*	*Height of Helicopter h*	*Time Taken to Hit the Helicopter T*	*Angle of Projection in Degree*	*Distance Travelled by the Helicopter during Time of Hit (AC)*
1	70	2.5	900	1500	900	2.1026	29.1476	152.708
2	70	2.5	900	1500	1000	2.16402	31.6845	157.335
3	70	2.5	900	1500	1100	2.22985	34.0746	162.305
4	70	2.5	900	1500	1200	2.2997	36.3217	167.59
5	70	2.5	900	1500	1300	2.37322	38.4313	173.166
6	70	2.5	900	1500	1400	2.45009	40.4098	179.01
7	70	2.5	900	1500	1500	2.53	42.2641	185.101

Source: Authors' own.

Formulation of the Master Equation While the Drone/ Helicopter is Moving Towards the Observer with Acceleration

Let O be any point of projection of the shell, A be the position of the drone/helicopter when the shot was fired. The muzzle velocity (initial velocity) of the shell is along OD. Let the angle of projection be α. The drone/helicopter is flying at height *h* metres along the line AB with an initial observed speed $\mathbf{v_0}$ **m/s** and acceleration **f m/s²**. C is a point where the shell can hit the drone/helicopter, that is, provided both reach C at the same time (Figure 6).

The starting point of the shell is O and at the same time the position of the flying object is at A such that AB = *d*. Let the shell hit the drone/helicopter after time *t*.

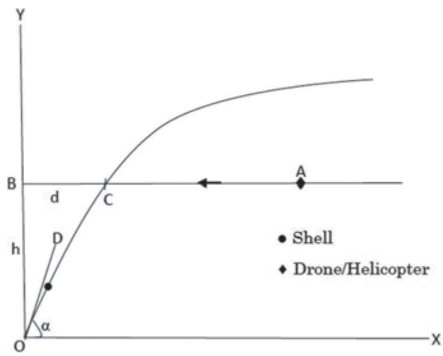

Figure 6 The Drone/Helicopter is Moving Towards the Observer with Acceleration *f*

Source: Authors' own.

The distance moved by the drone/helicopter in time *t*

$$AC = v_0.t + ½\, ft^2 \quad ...(10)$$

Horizontal distance moved by shell after time *t* is given by

$$BC = u \cos \alpha.t \quad ...(11)$$

In view of Equations (10) and (11), we have

$$BC = d - AC \Rightarrow u \cos \alpha.t = d - (v_0.t + ½\, ft^2)$$

and yields the following condition

$$½\, ft^2 + (u \cos \alpha + v_0).t - d = 0 \quad ...(12)$$

Equation (12) yields the expression of *t* as

$$t = \frac{-(u \cos \alpha + v_0) + \sqrt{(u \cos \alpha + v_0)^2 + 2fd}}{f} \quad \text{if } f \neq 0 \quad ...(13)$$

$$= \frac{d}{u \cos \alpha + v_0}, f = 0 \quad ...(14)$$

Vertical motion of shell from O to C is given by

$$h = u \sin \alpha . t - \tfrac{1}{2} g t^2 \qquad \text{...(15)}$$

Using the Equations (13), (14) and (15), we get

$$\Rightarrow 4.905 \left(\frac{-(u \cos \alpha + v_0) + \sqrt{(u \cos \alpha + v_0)^2 + 2fd}}{f} \right)^2 - \frac{-(u \cos \alpha + v_0) + \sqrt{(u \cos \alpha + v_0)^2 + 2fd}}{f} u \sin \alpha + h = 0$$

$$\text{if } f \neq 0 \qquad \text{...(16)}$$

$$4.905 \left(\frac{d}{(u \cos \alpha - v_0)} \right)^2 - \frac{d\, u \sin \alpha}{(u \cos \alpha - v_0)} + h = 0, \ \text{if } f = 0 \qquad \text{...(17)}$$

The angle of projection can be obtained by using the Equations (16) and (17).

Evaluation of Various Requisite Parameters

We calculated various parameters such as the angle of projection, time of hit and distance covered by the drone/helicopter at the time of collision by considering the drone speed as 20m/s, helicopter speed as 70m/s, speed of bullet as 900m/s and detected that the drone and helicopter are flying at height *h* (= 500m), horizontal distance *d* (= 800m) for various values of *f* (Tables 5, 6 and Figures 7 and 8). Through the master equations, we also estimated the same parameters for various values of

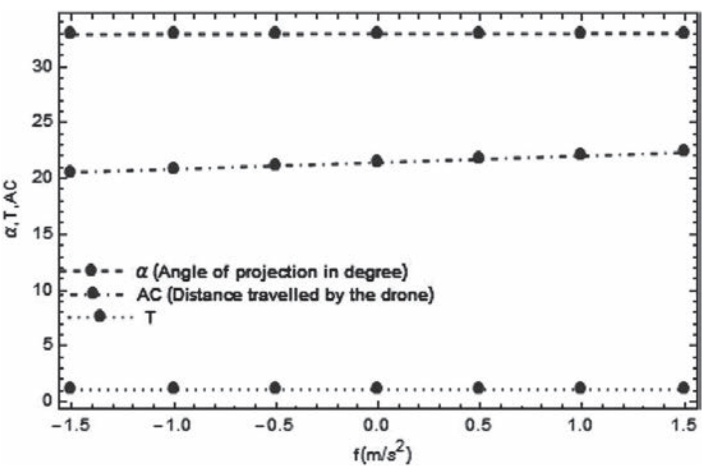

Figure 7 Variation of α, T, AC with *f* While the Drone is Moving Towards the Observer When *h* and *d* are Fixed
Source: Authors' own.

height (h = 900m to 1500m) of drone for f = 0.5 m/s^2 and helicopter for f = 2.5 m/s^2 (Tables 7, 8 and Figures 9 and 10) when the flying objects are moving towards the observer. The trends of time taken to hit the drone/helicopter (T), angle of projection α and distance travelled by the drone/helicopter during the time of hit (AC) can be seen in Figures 7, 8, 9, and 10.

Table 5 The Values of Angle of Projection, Time of Hit and Distance Covered by the Drone Moving Towards the Observer during the Time of Hit When *h* and *d* are Fixed

S. No.	*Speed of Drone m/s (v_0)*	*Acceleration/Retardation f m/s^2*	*Muzzle Velocity of Weapon m/s u*	*Horizontal Distance in Metre d*	*Height of Drone h*	*Time Taken to Hit the Drone T*	*Angle of Projection in Degree*	*Distance Travelled by the Drone during Time of Hit (AC)*
1	20	–1.5	900	800	500	1.03273	32.9276	19.8546
2	20	–1	900	800	500	1.03248	32.9362	20.1166
3	20	–0.5	900	800	500	1.03224	32.9449	20.3783
4	20	0	900	800	500	1.03199	32.9535	20.6398
5	20	0.5	900	800	500	1.03175	32.9622	20.901
6	20	1	900	800	500	1.0315	32.9708	21.162
7	20	1.5	900	800	500	1.03126	32.9794	21.4227

Source: Authors' own.

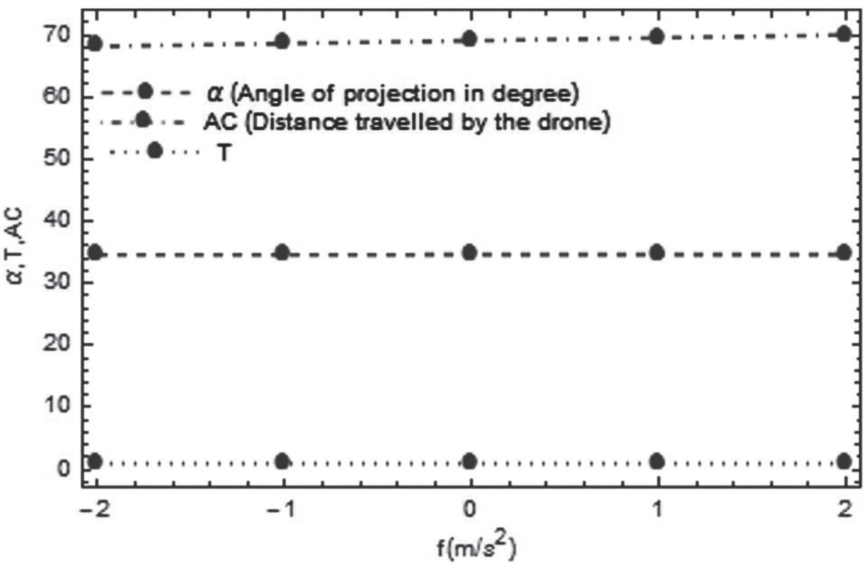

Figure 8 Variation of α, *T*, *AC* with *f* while the helicopter is moving towards the observer when *h* and *d* are fixed.
Source: Authors' own.

Table 6 Values of the Angle of Projection, Time of Hit and Distance Covered by the Helicopter Moving Towards the Observer during the Time of Collision When *h* and *d* are Fixed

S. No.	*Speed of Helicopter m/s (v_0)*	*Acceleration/ Retardation f m/s²*	*Muzzle Velocity of Weapon m/s u*	*Horizontal Distance in Metre d*	*Height of Helicopter h*	*Time Taken to Hit the Helicopter T*	*Angle of Projection in Degree*	*Distance Travelled by the Helicopter during Time of Hit (AC)*
1	70	–2.0	900	800	500	0.987816	34.5964	68.1713
2	70	–1	900	800	500	0.987394	34.6129	68.6301
3	70	0	900	800	500	0.986973	34.6295	69.0881
4	70	1	900	800	500	0.986552	34.6461	69.5453
5	70	2	900	800	500	0.986132	34.6626	70.0017

Source: Authors' own.

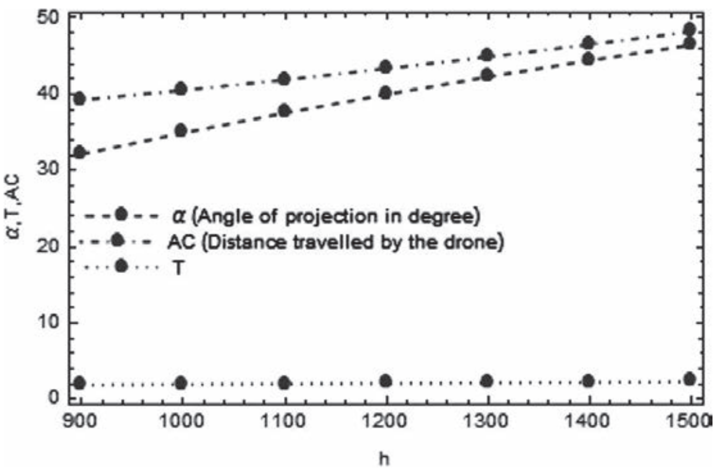

Figure 9 Variation of , *T*, *AC* with *h* while the drone is moving towards the observer when *f* and *d* are fixed.

Source: Authors' own.

Table 7 Values of Angle of Projection, Time of Hit and Distance Covered by the Drone Moving Towards the Observer during the Time of Collision When *f* and *d* are Fixed

S. No.	*Speed of Drone m/s (v_0)*	*Acceleration/ Retardation f m/s²*	*Muzzle Velocity of Weapon m/s u*	*Horizontal Distance in Metre d*	*Height of Drone h*	*Time Taken to Hit the Drone T*	*Angle of Projection in Degree*	*Distance Travelled by the Drone during Time of Hit (AC)*
1	20	0.5	900	1500	900	1.91696	32.1479	39.2579
2	20	0.5	900	1500	1000	1.9779	34.9278	40.536
3	20	0.5	900	1500	1100	2.04321	37.5406	41.9078
4	20	0.5	900	1500	1200	2.11251	39.9914	43.3659

S. No.	*Speed of Drone m/s (v_0)*	*Acceleration/ Retardation f m/s²*	*Muzzle Velocity of Weapon m/s u*	*Horizontal Distance in Metre d*	*Height of Drone h*	*Time Taken to Hit the Drone T*	*Angle of Projection in Degree*	*Distance Travelled by the Drone during Time of Hit (AC)*
5	20	0.5	900	1500	1300	2.18546	42.2869	44.9033
6	20	0.5	900	1500	1400	2.26173	44.4348	46.5135
7	20	0.5	900	1500	1500	2.34102	46.4437	48.1906

Source: Authors' own.

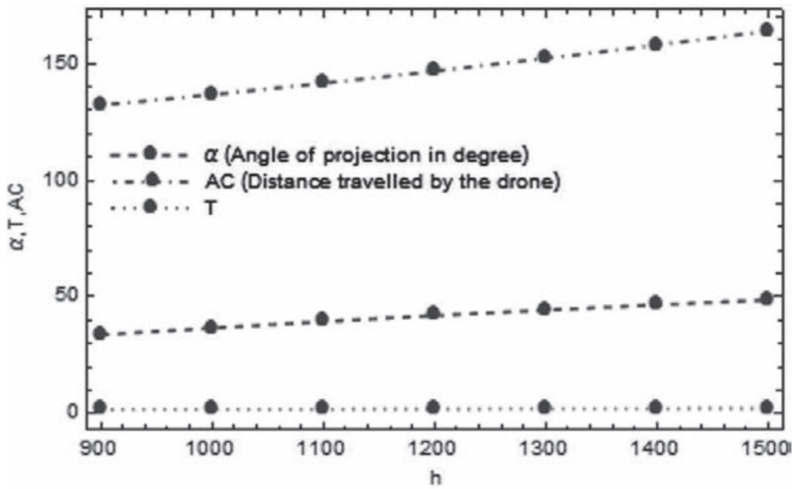

Figure 10 Variation of , *T*, *AC* with *h* While the Helicopter is Moving Towards the Observer When *f* and *d* are Fixed

Source: Authors' own.

Table 8 Values of the Angle of Projection, Time of Hit and Distance Covered by the Helicopter Moving Towards the Observer during the Time of Collision When *f* and *d* are Fixed

S. No.	*Speed of Helicopter m/s (v_0)*	*Acceleration/ Retardation f m/s²*	*Muzzle Velocity of Weapon m/s u*	*Horizontal Distance in Metre d*	*Height of Helicopter h*	*Time Taken to Hit the Helicopter T*	*Angle of Projection in Degree*	*Distance Travelled by the Helicopter during Time of Hit (AC)*
1	70	2.5	900	1500	900	1.82932	33.8224	132.235
2	70	2.5	900	1500	1000	1.89011	36.738	136.773
3	70	2.5	900	1500	1100	1.95528	39.4752	141.648
4	70	2.5	900	1500	1200	2.02444	42.0399	146.834
5	70	2.5	900	1500	1300	2.09727	44.4394	152.307
6	70	2.5	900	1500	1400	2.17342	46.6824	158.044
7	70	2.5	900	1500	1500	2.2526	48.778	164.025

Source: Authors' own.

Conclusion

Nowadays, slow moving objects such as drones pose an immense threat to the internal and external security of all countries. Therefore, we have tried to introduce a mathematical/scientific conjecture to weapon technocrats to develop an automatic weapon to shoot down slow moving objects. This also paves the way for any defence personnel to understand this concept.

Hypothetically, we can engineer such automatic mechanical weapons to shoot down flying objects if the firing weapon is augmented with sensors/detectors observing parameters such as the velocity, position and manoeuvring effect of the drone, and accordingly the up-down movement of the barrel of the weapon can be programmed with the master equations so developed. It is also concluded that even with the manoeuvring effect $-L/2$ to $+L/2$ where L be the length of the drone/helicopter in metres, the target will be shot down with the same angle of projection as calculated for no manoeuvring effect.

Acknowledgements

The authors are grateful to Maj Gen Sanjeev Dogra, the Deputy Commandant, NDA, for providing valuable inputs. N. Pant is also thankful to Dr S. Faruqi, NDA, for his inputs. The authors are grateful to the esteemed reviewers for a rigorous review of the commentary.

Notes

1. By flying objects, we mean Drones/helicopters/aircraft.
2. Amitabha Ghosh, *Introduction to Dynamics*, Singapore: Springer Singapore, 2018; Friedrich Pfeiffer and Thorsten Schindler, *Introduction to Dynamics*, Heidelberg: Springer Berlin, 2015; Neeraj Pant and A.N. Srivastava, *Dynamics for Undergraduates*, CBS Publisher and Distributors P Ltd, New Delhi, 2011; M. Ray and G.C. Sharma, *A Textbook on Dynamics*, S. Chand and Company Ltd., Noida (UP) 2006.
3. If f is a retardation then we will consider Negative sign (–).
4. All figures and tabular values in this commentary are plotted/calculated using Microsoft Word and Mathematica software.